Int. J. Heat Mass Transfer. Vol. 22, pp. 1323–1329
Pergamon Press Ltd. 1979. Printed in Great Britain

0017-9310/79/1001-1323 $02.00/0

LAMINAR BOUNDARY-LAYER FLOW OF NON-NEWTONIAN FLUID

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(*Received* 30 *October* 1978 *and in revised form* 8 *February* 1979)

Abstract—A solution for the two-dimensional and axisymmetric laminar boundary-layer momentum equation of power-law non-Newtonian fluid is presented. The analysis makes use of the Merk–Chao series solution method originally devised for the flow of Newtonian fluid. The universal functions for the leading term in the series are tabulated for n from 0.2 to 2. Equations governing the universal functions associated with the second and the third terms are provided. The solution together with either Lighthill's formula or Chao's formula constitutes a simple yet general procedure for the calculation of wall shear and surface heat transfer rate. The theory was applied to flows over a circular cylinder and a sphere and the results compared with published data.

NOMENCLATURE

- f, dimensionless stream function, equation (8);
- f_0, f_1, f_2, Universal functions, equation (15);
- K, n, parameters in the power-law model, equation (3);
- L, characteristic length;
- R, radius of a circular cylinder or a sphere;
- R_e, Reynolds number, $= \rho L^n U_\infty^{2-n}/K$;
- r, distance from a point on the surface to the axis of symmetry, $r = L$ for two-dimensional problems;
- U, velocity component outside the boundary layer;
- U_∞, characteristic velocity;
- u, velocity component in x-direction;
- v, velocity component in y-direction;
- x, distance along the surface from the stagnation point or the leading edge;
- y, distance normal to x.

Greek symbols

- β, wedge variable, equation (12);
- ρ, density;
- ψ, stream function, equation (6);
- ξ, transformed x-coordinate, equation (7a);
- η, transformed y-coordinate, equation (7b);
- τ_{xy}, shear stress;
- τ_w, wall shear stress.

1. INTRODUCTION

THE NEED for studying boundary layer transfer of non-Newtonian fluid was realized about two decades ago. This was probably caused by the growing use of non-Newtonian fluids such as molten plastics, polymers, foods, etc., in various manufacturing and processing industries. In most technological applications, it is often the wall shear and the surface heat transfer rate that are of the greatest interest. Since the validity of boundary layer energy equation is not restricted to any particular class of fluid, the heat transfer formula of Lighthill [1] or of Chao [2] can also be used for flow of non-Newtonian fluid. Because of large Prandtl number for most non-Newtonian fluids, the Lighthill formula is generally accurate enough for most engineering applications. However, the Chao's formula can be used if more accurate results are desired. Since both formulas require that the wall shear be known, the need for a simple yet general procedure for calculating the wall shear is obviously called for in the heat transfer calculation.

To provide such a procedure, the Karman–Pohlhausen integral method originally devised for Newtonian fluid was used by Acrivos *et al.* [3] and by Bizzell and Slattery [4]. Acrivos *et al.* found that the method is substantially less accurate for power-law fluids than for Newtonian substances under otherwise same flow conditions. For this reason, they presented [5] another approximate solution by an asymptotic technique. Their solution, in essence, is an interpolation between two limiting solutions. This solution is simple to use, yet it is not clear how the calculations are to be improved if more accurate results are desired. The Blasius series method for analyzing the boundary layer equation of Newtonian fluid was extended to the power-law fluids by Wolf and Szewczyk [6]. They pointed out that very little can be expected from the series for n

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greater than about 1.2 for the entire region of laminar flow. The Goertler series method for solving the boundary layer momentum equation was extended to power-law non-Newtonian fluid by Serth and Kiser [7]. With the universal functions they provided, the analysis appears to be of limited usefulness for pseudo-plastic fluids ($n < 1$).

A rapid computational procedure which makes use of universal function for calculating boundary layer transfer in flow of Newtonian fluid was first proposed by Merk [8] and later corrected by Chao and Fagbenle [9]. This series solution method is able to provide accurate results for flows of Newtonian fluids even if only the first term of the series is used [10, 11]. The first term of the series represents the local similarity solution. The remaining provides a rigorous correction for the departure from local similarity. In this paper we shall extend the Merk–Chao method to the solution of the laminar boundary-layer momentum equation of non-Newtonian fluid. The flow is assumed to be steady and incompressible.

2. ANALYSIS

For a steady, incompressible, laminar flow past a smooth axisymmetric object, the boundary-layer equations are [12]

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\mathrm{d}U}{\mathrm{d}x}+\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}. \tag{2}$$

For flow past a two-dimensional body, one needs only to set $r = L$, L being a reference length. For the so-called power-law fluids, one has:

$$\tau_{xy}=K\left(\frac{\partial u}{\partial y}\right)^n, \tag{3}$$

in which K and n are material constants of the fluid and are both positive. The appropriate boundary conditions are:

$$u(x,0)=v(x,0)=0 \tag{4a,b}$$

$$u(x,\infty)=U(x). \tag{5}$$

Upon introducing a stream function $\psi(x, y)$ defined by

$$ru=\frac{\partial\psi}{\partial y},\quad rv=-\frac{\partial\psi}{\partial x}, \tag{6a,b}$$

the continuity equation (1) is then identically satisfied. By considering the transformation

$$\left.\begin{matrix} x \\ \\ y\end{matrix}\right\}\rightarrow\left\{\begin{aligned}&\xi=\frac{2}{L}\int_0^x\left(\frac{r}{L}\right)^{n+1}\left(\frac{U}{U_\infty}\right)^{2n-1}\mathrm{d}x && \text{(7a)}\\ &\eta=R_e^{1/(n+1)}\frac{r}{L}\frac{U}{U_\infty}\xi^{-1/(n+1)}\frac{y}{L}, && \text{(7b)}\end{aligned}\right.$$

and introducing a dimensionless stream function $f(\xi,\eta)$ such that

$$\psi(x,y)=L^2U_\infty R_e^{-1/(n+1)}\xi^{1/(n+1)}f(\xi,\eta), \tag{8}$$

the momentum equation (2) becomes:

$$\frac{\partial^3 f}{\partial\eta^3}+\frac{2}{n(n+1)}f\left(\frac{\partial^2 f}{\partial\eta^2}\right)^{2-n}+\beta\frac{1}{n}\left(\frac{\partial^2 f}{\partial\eta^2}\right)^{1-n}\left[1-\left(\frac{\partial f}{\partial\eta}\right)^2\right]$$
$$=\frac{2\xi}{n}\left(\frac{\partial^2 f}{\partial\eta^2}\right)^{1-n}\left[\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta}-\frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2}\right]. \tag{9}$$

The associated boundary conditions are:

$$f(\xi,0)=0,\quad \frac{\partial f}{\partial\eta}(\xi,0)=0 \tag{10a,b}$$

$$\frac{\partial f}{\partial\eta}(\xi,\infty)=1. \tag{11}$$

Here $R_e=\rho L^n U_\infty^{2-n}/K$ is the generalized Reynolds number and

$$\beta=\frac{L}{U_\infty}\frac{\xi\dfrac{\mathrm{d}U}{\mathrm{d}x}}{\left(\dfrac{r}{L}\right)^{n+1}\left(\dfrac{U}{U_\infty}\right)^{2n}} \tag{12}$$

is the so-called "wedge variable". The x-component of velocity and the wall shear are, respectively, given by

$$u=U\frac{\partial f}{\partial\eta}, \tag{13}$$

and

$$\tau_w=\frac{KU_\infty^n}{L^n}R_e^{n/(n+1)}\left(\frac{r}{L}\right)^n\left(\frac{U}{U_\infty}\right)^{2n}\times\xi^{-n/(n+1)}\left[\frac{\partial^2 f}{\partial\eta^2}(\xi,0)\right]^n. \tag{14}$$

According to Merk and Chao, the appropriate series solutions for equation (9) satisfying equations (10a,b) and (11) is:

$$f(\xi,\eta)=f_0(\beta,\eta)+\xi\frac{\mathrm{d}\beta}{\mathrm{d}\xi}f_1(\beta,\eta)+\xi^2\frac{\mathrm{d}^2\beta}{\mathrm{d}\xi^2}f_2(\beta,\eta)+\ldots \tag{15}$$

Upon substituting equation (15) into equation (9), followed by first collecting terms free of derivatives of β and then terms common to $\xi(\mathrm{d}\beta/\mathrm{d}\xi)$, $\xi^2(\mathrm{d}^2\beta/\mathrm{d}\xi^2)$, etc., one obtains a sequence of differential equations. The first equation is, with prime denoting differentiation with respect to η,

$$f_0'''+\frac{2}{n(n+1)}f_0(f_0'')^{2-n}+\frac{\beta}{n}[1-(f_0')^2](f_0'')^{1-n}=0, \tag{16}$$

with

$$f_0(\beta,0)=f_0'(\beta,0)=0 \tag{17a,b}$$

$$f_0'(\beta,\infty)=1. \tag{18}$$

It should be noted that β is a function of x only. At a given streamwise location, it is fixed. Hence, equation (16) can be integrated as if it were an ordinary differential equation. The f_0 function is universal in the sense that for a given β and n, it may be evaluated once and for all.

The second and third equations are

$$f_1''' + \left[\frac{2(2-n)}{n(n+1)} f_0(f_0'')^{1-n} + \frac{1-n}{n}\beta(f_0'')^{-n} - \frac{1-n}{n}\beta(f_0'')^{-n}(f_0')^2\right] f_1'' - \frac{2}{n}(\beta+1) f_0'(f_0'')^{1-n} f_1' + \frac{n+2}{n+1}\frac{2}{n}(f_0'')^{2-n} f_1 = \frac{2}{n}\left[f_0'(f_0'')^{1-n}\frac{\partial f_0'}{\partial\beta} - (f_0'')^{2-n}\frac{\partial f_0}{\partial\beta}\right], \quad (19)$$

and

$$f_2''' + \left[\frac{2(2-n)}{n(n+1)} f_0(f_0'')^{1-n} + \frac{1-n}{n}\beta(f_0'')^{-n} - \frac{1-n}{n}\beta(f_0'')^{-n}(f_0')^2\right] f_2'' - \frac{2}{n}(\beta+2) \times f_0'(f_0'')^{1-n} f_2' + \left[\frac{2}{n(n+1)} + \frac{4}{n}\right](f_0'')^{2-n} f_2 = \frac{2}{n}\left[f_0'(f_0'')^{1-n} f_1' - (f_0'')^{2-n} f_1\right], \quad (20)$$

with

$$f_i(\beta, 0) = f_i'(\beta, 0) = f_i'(\beta, \infty) = 0, \quad (21\text{a,b,c})$$

where $i = 1$ or 2. Like f_0, f_i's are also universal. The derivative $\partial^2 f/\partial\eta^2$ evaluated at the wall is:

$$\frac{\partial^2 f}{\partial \eta^2}(\xi, 0) = f_0''(\beta, 0) + \xi \frac{d\beta}{d\xi} f_1''(\beta, 0) + \xi^2 \frac{d^2\beta}{d\xi^2} f_2''(\beta, 0) + \ldots. \quad (22)$$

It is fitting to point out that when β is a constant, the flow is similar and the series expansion (15) has only the first term. The expression for wall shear now becomes:

$$\tau_w = \frac{K U_\infty^n}{L^n} R_e^{n/(n+1)} \left(\frac{r}{L}\right)^n \left(\frac{U}{U_\infty}\right)^{2n} \times \xi^{-n/(n+1)} [f_0''(\beta, 0)]^n. \quad (23)$$

When the flow is nonsimilar, the first term, f_0, in (15) constitutes the local similarity solution. The remaining terms, taken collectively, may be regarded as corrections for the deviation from local similarity. Hence equation (23) also represents the local similar solution of a nonsimilar problem. We reiterate that if one sets $r = L$, all equations presented in this paper are then applicable to two-dimensional flows.

Once the wall shear function is calculated, the formula for surface heat transfer rate calculation is readily available. The readers are referred to [1, 2, 13].

3. APPLICATIONS

We now consider a flow of power-law fluid past a wedge for which $U(x)$ is proportional to x^m. It can then be demonstrated that:

$$\beta = 2m/[m(2n-1)+1], \quad \text{a constant} \quad (24\text{a})$$

$$\xi = \frac{2}{m(2n-1)+1} \cdot \frac{x}{L}\left(\frac{U}{U_\infty}\right)^{2n-1}, \quad (24\text{b})$$

and

$$\tau_w = \frac{K U_\infty^n}{L^n} R_e^{n/(n+1)} \left(\frac{U}{U_\infty}\right)^{2n} \times \xi^{-n/(n+1)} [f_0''(\beta, 0)]^n. \quad (24\text{c})$$

Since β is a constant, the flow is similar. This agrees with the work of Hsu and Cothern [14]. Comparing equation (24c) with equation (12) in [14], one has:

$$f_0''(\beta, 0) = \left[\frac{2}{n(n+1)}\right]^{1/(n+1)} \cdot [f_{\eta\eta}(0)]_{\text{H-C}}, \quad (25)$$

where $[f_{\eta\eta}(0)]_{\text{H-C}}$ denotes the wall derivative $f_{\eta\eta}(0)$ in [14].

By Meksyn's method, Hsu and Cothern [14] have calculated values of $[f_{\eta\eta}(0)]_{\text{H-C}}$ for wedge angle from 0 to π and n from 0.2 to 2. Using their values of $[f_{\eta\eta}(0)]_{\text{H-C}}$ and equation (25), the value of universal function, $f_0''(\beta, 0)$, can be obtained. They are tabulated in Tables 1 and 2. With these, the wall shear can be calculated from (23).

We next consider flow across a long horizontal circular cylinder. According to Shah *et al.* [15], the velocity profile for $x/R \leqslant 1.05$ and for $0.6 \leqslant n \leqslant 1$ is:

$$\frac{U}{U_\infty} = 0.92\frac{x}{R} - 0.131\left(\frac{x}{R}\right)^3. \quad (26)$$

Using this expression, Serth and Kiser [7] have reported numerical data of the dimensionless wall velocity gradient for several values of n. To effect a direct comparison, we shall also use equation (26) and assume that it is valid for all n and for the entire flow region. With the velocity profile specified, ξ and β can be evaluated from (7a) and (12). With the aid of Table 1 or 2, the dimensionless wall velocity gradient can then be calculated from equation (23). The results are, along with the data reported in [7], displayed in Fig. 1. It is seen that they compared quite well except for $n = 0.2$. This may be due to the fact that Serth and Kiser [7] had difficulty in obtaining sufficient universal functions for values of $n \leqslant 0.5$. Also shown in Fig. 1 are the results for $n = 0.4$ and 1.6 calculated from the 2-term Blasius series in [6] and with the velocity profile represented by equation (26). Good agreement is seen for x/R up to about 1. Since the third term in the Blasius series is positive for $n > 1$ and negative for $n < 1$, better agreement is expected if the results from 3-term Blasius series were compared. It is interesting to point out that, unlike the case of Newtonian fluids, the first term of the Blasius series is monotonically increasing with x.

Table 1. Value of $f_0''(\beta, 0)$ for pseudoplastic fluids

$n = 0.200$		$n = 0.300$		$n = 0.400$		$n = 0.500$	
β	$f_0''(\beta, 0)$	β	$f_0''(\beta, 0)$	β	$f_0''(\beta, 0)$	β	$f_0''(\beta, 0)$
0.000	0.895	0.000	0.675	0.000	0.578	0.000	0.529
0.109	1.457	0.108	1.034	0.106	0.842	0.105	0.738
0.238	1.844	0.233	1.280	0.227	1.030	0.222	0.894
0.395	2.236	0.380	1.663	0.366	1.234	0.353	1.058
0.588	2.762	0.556	1.892	0.526	1.502	0.500	1.261
0.833	3.418	0.769	2.258	0.714	1.733	0.667	1.436
1.154	4.132	1.034	2.669	0.937	2.001	0.857	1.631
1.591	5.127	1.373	3.179	1.207	2.316	1.077	1.846
2.222	6.561	1.818	3.837	1.538	2.689	1.333	2.094
3.214	8.767	2.432	4.716	1.957	3.143	1.636	2.356
5.000	12.607	3.333	5.947	2.500	3.704	2.000	2.667
$n = 0.600$		$n = 0.700$		$n = 0.800$		$n = 0.900$	
β	$f_0''(\beta, 0)$	β	$f_0''(\beta, 0)$	β	$f_0''(\beta, 0)$	β	$f_0''(\beta, 0)$
0.000	0.500	0.000	0.485	0.000	0.475	0.000	0.471
0.104	0.676	0.103	0.638	0.102	0.613	0.101	0.595
0.217	0.813	0.213	0.761	0.208	0.727	0.204	0.703
0.341	0.951	0.330	0.882	0.319	0.834	0.309	0.801
0.476	1.112	0.455	1.012	0.435	0.943	0.417	0.889
0.625	1.253	0.588	1.128	0.556	1.051	0.526	0.976
0.789	1.400	0.732	1.246	0.682	1.138	0.638	1.058
0.972	1.557	0.886	1.365	0.814	1.233	0.753	1.133
1.176	1.734	1.053	1.490	0.952	1.327	0.870	1.212
1.406	1.903	1.233	1.616	1.098	1.423	0.989	1.287
1.667	2.085	1.429	1.749	1.250	1.518	1.111	1.357

Table 2. Value of $f_0''(\beta, 0)$ for dilatant fluids

$n = 1.000$		$n = 1.200$		$n = 1.400$		$n = 1.600$	
β	$f_0''(\beta, 0)$	β	$f_0''(\beta, 0)$	β	$f_0''(\beta, 0)$	β	$f_0''(\beta, 0)$
0.000	0.469	0.000	0.472	0.000	0.476	0.000	0.488
0.100	0.588	0.098	0.580	0.096	0.574	0.094	0.587
0.200	0.687	0.192	0.669	0.185	0.654	0.179	0.659
0.300	0.775	0.283	0.742	0.268	0.704	0.254	0.714
0.400	0.856	0.370	0.807	0.345	0.754	0.323	0.756
0.500	0.930	0.455	0.864	0.417	0.819	0.385	0.795
0.600	0.997	0.536	0.916	0.484	0.864	0.441	0.828
0.700	1.063	0.614	0.963	0.547	0.898	0.493	0.857
0.800	1.124	0.690	1.006	0.606	0.931	0.541	0.881
0.900	1.184	0.763	1.047	0.662	0.961	0.584	0.905
1.000	1.233	0.833	1.079	0.714	0.987	0.625	0.925

$n = 1.800$		$n = 2.000$	
β	$f_0''(\beta, 0)$	β	$f_0''(\beta, 0)$
0.000	0.490	0.000	0.498
0.093	0.592	0.091	0.605
0.172	0.658	0.167	0.652
0.242	0.707	0.231	0.700
0.303	0.744	0.286	0.736
0.357	0.777	0.333	0.765
0.405	0.805	0.375	0.789
0.449	0.828	0.412	0.808
0.488	0.848	0.444	0.823
0.523	0.866	0.474	0.838
0.556	0.882	0.500	0.851

We finally present the wall shear function for flow past a sphere for which

$$r/R = \sin(x/R). \tag{27}$$

If we assume that the external velocity distribution is independent of the degree of deviation of the fluid from Newtonian behavior, we have, according to potential theory,

$$\frac{U}{U_\infty} = \tfrac{3}{2}\sin(x/R). \tag{28}$$

Thus, the wall shear can be calculated from equation (23). They are displayed in Fig. 2 and are self-explanatory.

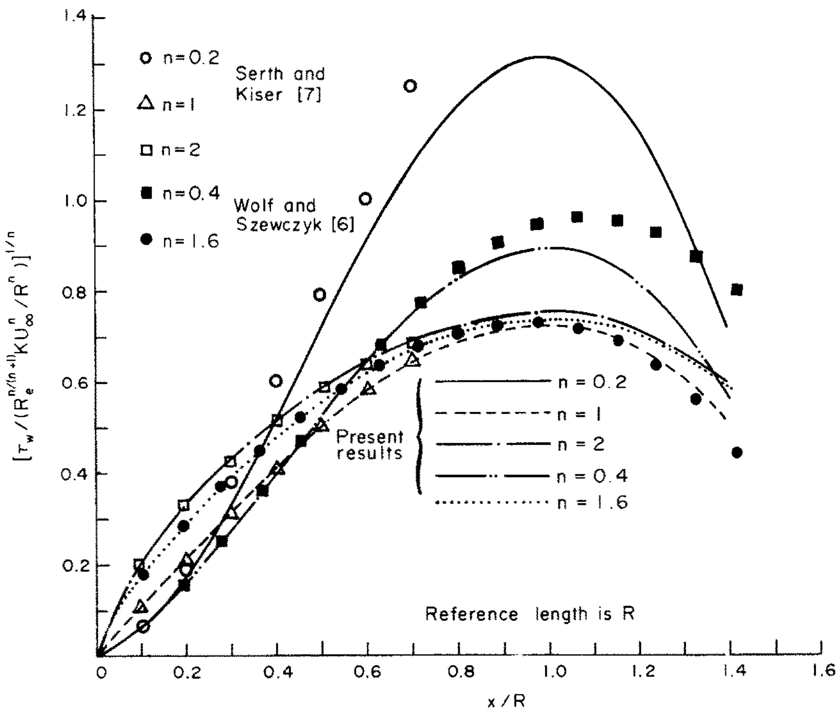

FIG. 1. Wall shear for flow over a horizontal circular cylinder.

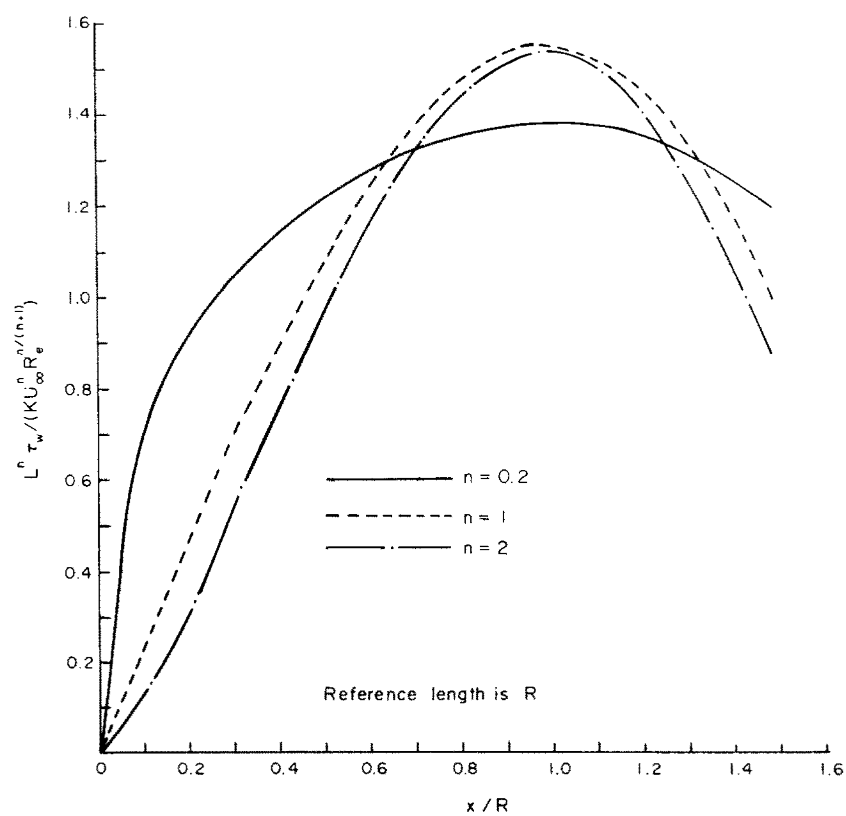

FIG. 2. Wall shear for flow past a sphere.

4. CONCLUDING REMARKS

The Merk–Chao series solution method has been extended to the solution of boundary-layer momentum equation of power-law non-Newtonian fluid. Equation (23) together with Tables 1 and 2 provide a simple yet general routine procedure for determining the wall shear over two-dimensional and axisymmetric bodies of arbitrary contour. With the wall shear function calculated, the local surface heat transfer rate can be evaluated from [1] or [2]. In addition, the thermal response behavior of boundary layer flow of the power-law fluid can also be investigated [16].

Acknowledgement– Special thanks is given to Mrs. Grace Reger of the Planning Research Corporation, Fluid System Design Section, for her skillful preparation of the manuscript.

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ECOULEMENT AVEC COUCHE LIMITE LAMINAIRE D'UN FLUIDE NON-NEWTONIEN

Résumé—On présente une solution de l'équation de la couche limite laminaire bidimensionnelle et axisymétrique pour un fluide non-newtonien à loi puissance. L'analyse utilise la méthode de Merk–Chao imaginée pour les fluides newtoniens. Les fonctions universelles pour le terme principal de la série sont tabulées pour n compris entre 0,2 et 2. On donne les équations reliant les fonctions universelles aux second et troisième termes. La solution avec soit la formule de Lighthill soit la formule de Chao, constitue une procédure simple et générale pour le calcul de la tension et du flux thermique pariétaux. La théorie est appliquée aux écoulements autour du cylindre circulaire et de la sphère et les résultats sont comparés aux données publiées.

LAMINARE GRENZSCHICHTSTRÖMUNG EINER NICHT-NEWTONSCHEN FLÜSSIGKEIT

Zusammenfassung—Eine Lösung für die zweidimensionale und axialsymmetrische laminare Grenzschicht-Impulsgleichung einer dem Potenzgesetz gehorchenden nicht-newtonschen Flüssigkeit wird beschrieben. Bei der Analyse werden die Merk–Chao-Reihen verwendet, die ursprünglich für die Strömung einer newtonschen Flüssigkeit abgeleitet wurden. Die allgemeinen Funktionen für das erste Glied der Reihen werden für n von 0,2 bis 2 tabellarisch angegeben. Gleichungen, die die allgemeinen Funktionen für das zweite und dritte Glied bestimmen, werden angegeben. Die Lösung, entweder mit der Formel von Lighthill oder der von Chao zusammen, bildet eine einfache, aber allgemeine Methode für die Berechnung der Wandschubspannung und des Wärmeübergangs an der Oberfläche. Die Theorie wurde für die Strömung um einen Kreiszylinder und eine Kugel angewendet und die Ergebnisse mit bekannten Daten verglichen.

ТЕЧЕНИЕ НЕНЬЮТОНОВСКОЙ ЖИДКОСТИ В ЛАМИНАРНОМ ПОГРАНИЧНОМ СЛОЕ

Аннотация — Представлено решение уравнения количества движения для двухмерного и осесимметричного ламинарного пограничного слоя неньютоновской жидкости степенного типа. Используется метод разложения в ряды Мерка–Чао, предложенный первоначально для анализа течения ньютоновской жидкости. Представлена таблица универсальных функций в диапазоне $0{,}2 \leq n \leq 2$ для первого члена разложения. Приведены уравнения, определяющие универсальные функции, связанные со вторым и третьим членами разложения. Предложенное решение вместе с формулой Лайтхилла или формулой Чао представляет собой простой и достаточно общий метод расчета напряжения трения на стенке и интенсивности теплообмена на обтекаемой поверхности. Метод проверен на примере обтекания круглого цилиндра и сферы. Проведено сравнение полученных результатов с опубликованными данными.